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Trieste

# Introduction to Short Wavelength Coherent Light Sources: Present and Outlook

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## ☐ Coherence of Radiation

- Transverse and longitudinal coherence length
- Brilliance, degeneracy parameter

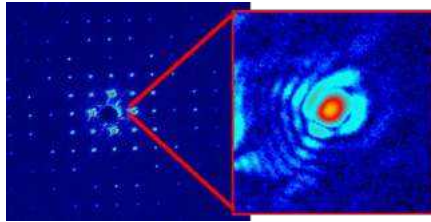
## ☐ Light Sources

- High Harmonic Generation (in gas)
- Free-Electron Laser (high gain regime)

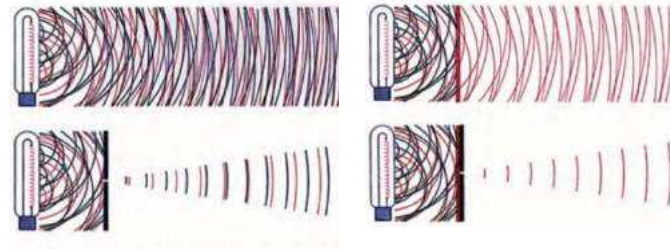
## ☐ Overview and Perspectives

# Coherence of Radiation

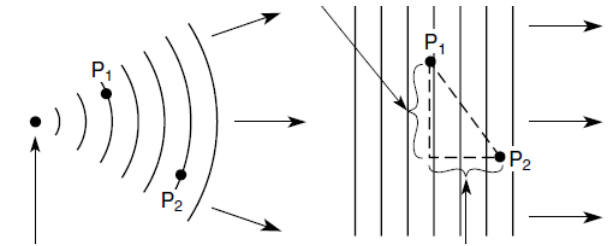
## Interference fringes



## Collimated, monochromatic light



## Correlated field (*Glauber's*)



### 1<sup>st</sup> order correlation function:

$$g_1(\vec{r}_1, t_1; \vec{r}_2, t_2) = \frac{\langle E^*(\vec{r}_1, t_1) E(\vec{r}_2, t_2) \rangle}{\sqrt{\langle |E(\vec{r}_1, t_1)|^2 \rangle \langle |E(\vec{r}_2, t_2)|^2 \rangle}}$$

### Visibility of fringe pattern:

$$v(\lambda) = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} |g_1(\vec{r}_1, t_1; \vec{r}_2, t_2)|$$

### Degree of transverse coherence:

$$\xi_c = \frac{\iint |g_1(\vec{r}_1, \vec{r}_2)|^2 \langle I(\vec{r}_1) \rangle \langle I(\vec{r}_2) \rangle d\vec{r}_1 d\vec{r}_2}{[\int \langle I(\vec{r}_1) \rangle d\vec{r}_1]^2}$$

### Longitudinal coherence length:

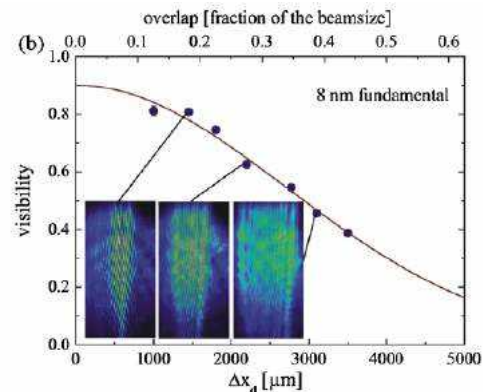
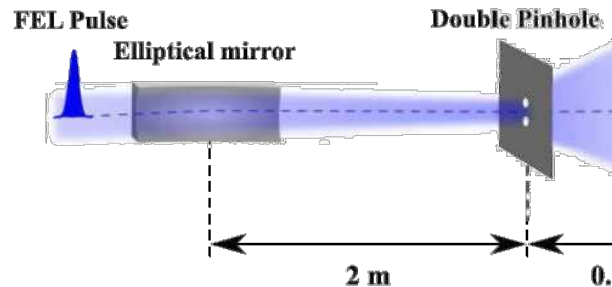
$$\tau_{c,rms} = \int_{-\infty}^{\infty} |g_1(\tau)|^2 d\tau$$

D. Attwood and A. Sakdinawat, *X-rays and Extreme Ultraviolet Radiation*, Cambridge Univ. Press (2016).

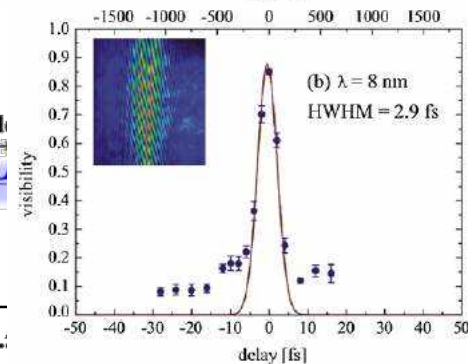
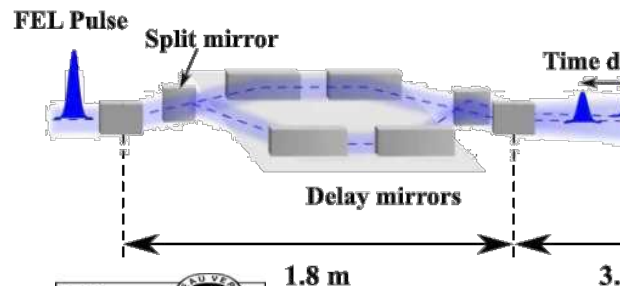
A. Singer et al., arXiv:1206.1091v1 (2012)

S. Roling et al., PRST-AB 14 (2011)

### (a) BL2



### (b) PG2



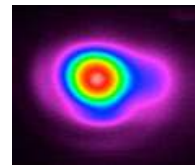
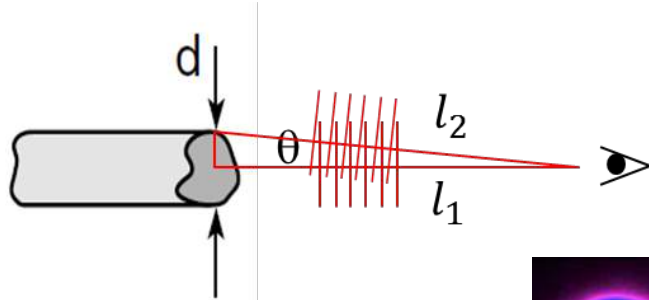
Detector



# Coherence Lengths

Classical model: path length over which two waves become **out of phase**

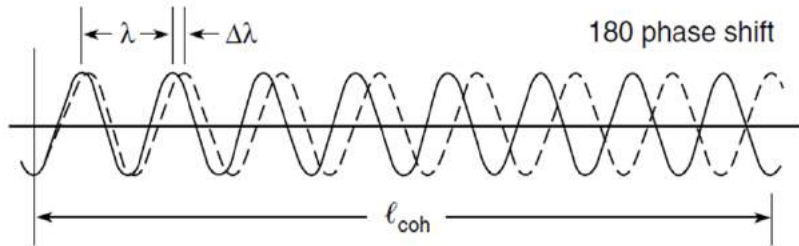
Uncertainty Principle: the smallest **phase space area** occupied by the light pulse



TEM00

Minimum transverse phase space area ("**emittance**") of a transversally coherent light pulse

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \quad \text{and} \quad \theta = \frac{\Delta p_x}{p_z} \cong \frac{\Delta p_x}{(h/\lambda)} \Rightarrow \frac{d}{2} \theta_c = \frac{\lambda}{4\pi}$$



$$\Delta t \Delta E \geq \frac{\hbar}{2} \rightarrow \frac{c \Delta t}{\lambda^2 / \Delta \lambda} \geq \frac{1}{4\pi} \Rightarrow L_{c,\parallel} = c \Delta t = \frac{1}{4\pi} \frac{\lambda^2}{\Delta \lambda}$$



Longitudinal coherence shows up  $\Delta t_{\text{pulse}} \leq L_c$

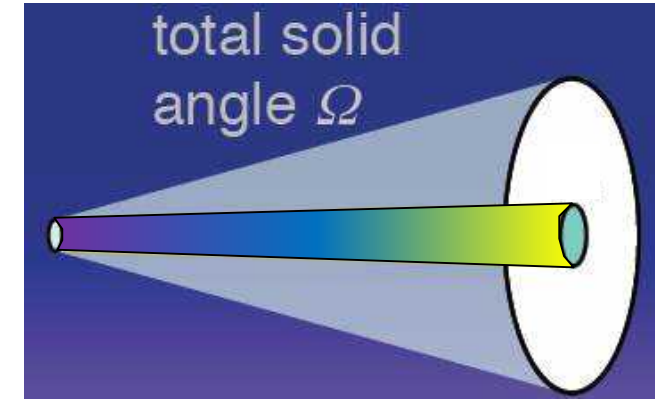




# Brilliance

- The **fraction of spectral flux transversally coherent**, emitted in a solid angle  $\Sigma_{x'}\Sigma_{y'} = \Omega$  from a source of size  $\Sigma_x\Sigma_y = (d/2)^2$ , is:

$$\left(\frac{dN_\gamma/dt}{\Delta\omega/\omega}\right)_{\perp,coh} = \left(\frac{dN_\gamma/dt}{\Delta\omega/\omega}\right) \frac{\theta_c^2}{\Omega} = \left(\frac{dN_\gamma/dt}{\Delta\omega/\omega}\right) \frac{\lambda^2}{(4\pi)^2 \left(\frac{d}{2}\right)^2 \Omega} = \mathbf{B} \times \left(\frac{\lambda}{2}\right)^2$$



Full transverse coherence for  $\frac{\theta_c^2}{\Omega} = 1$  or  $\Sigma_x\Sigma_y\Sigma_{x'}\Sigma_{y'} = \left(\frac{\lambda}{4\pi}\right)^2$ ,

**“diffraction limit”**

- The **number of photons transversally and longitudinally coherent** is:

$$n_{coh} = \left(\frac{dN_\gamma/dt}{\Delta\omega/\omega}\right)_{\perp,coh} \cdot \frac{L_{c,\parallel}}{c} \cdot \frac{\Delta\omega}{\omega} = B \left(\frac{\lambda}{2}\right)^2 \frac{\lambda^2}{2c\Delta\lambda} \frac{\Delta\lambda}{\lambda} = \frac{B\lambda^3}{8c}$$

**“degeneracy parameter”**

- *It is harder to get full coherence at shorter wavelengths*
- *In a real beamline, B (at sample)  $\propto$  B (at source)*



# Outline

## ☐ Coherence at Light Sources

- Transverse and longitudinal coherence length of radiation
- Brilliance, degeneracy parameter

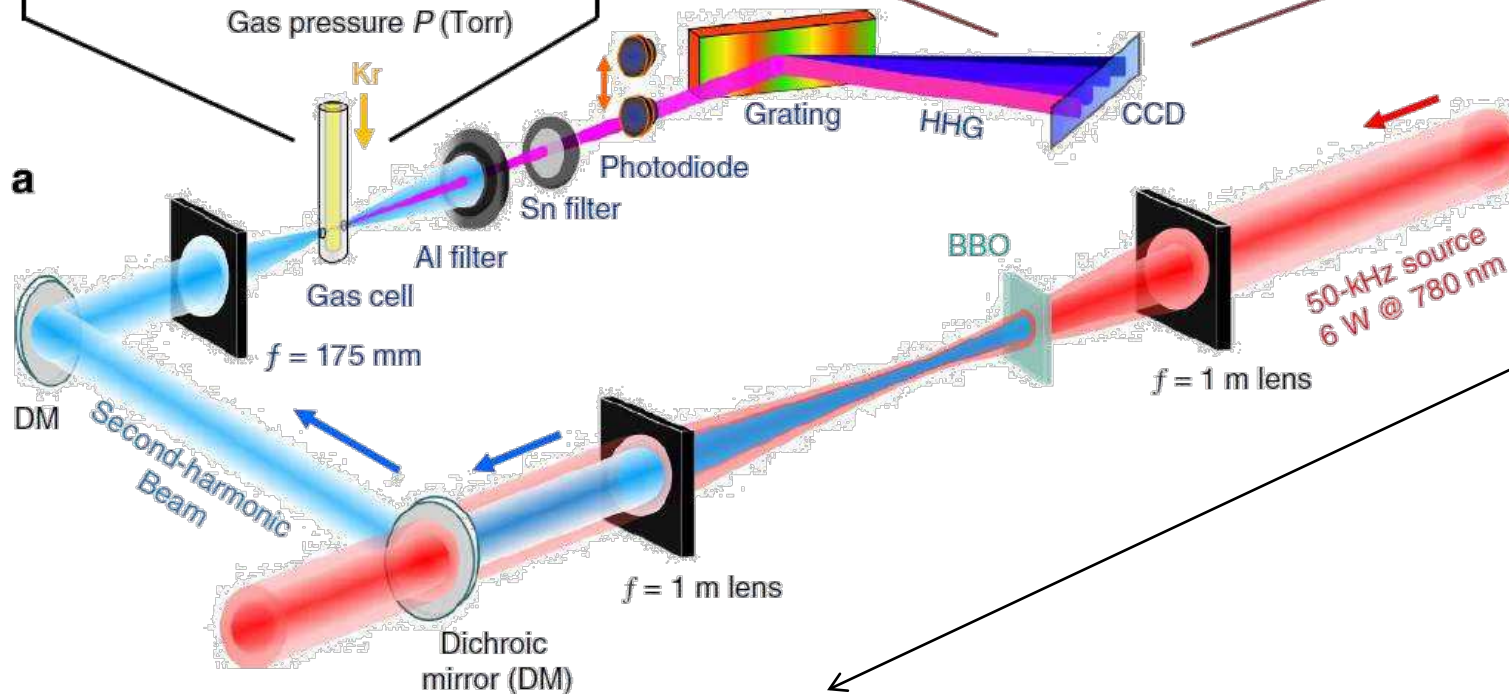
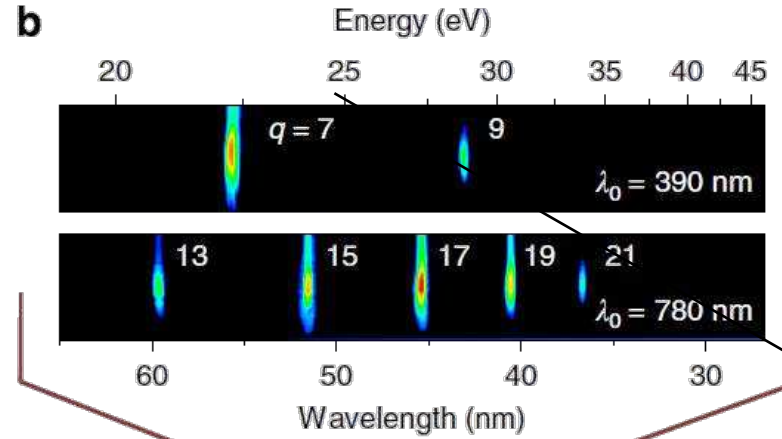
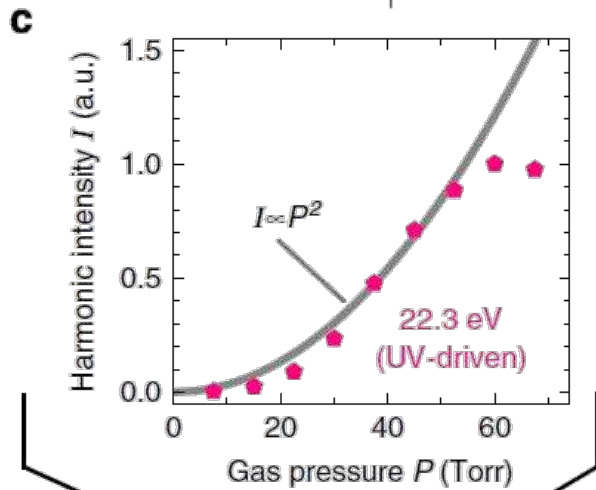
## ☐ Light Sources

- High Harmonic Generation (in gas)
- Free-Electron Laser (high gain regime)

## ☐ Overview and Perspectives



# HHG in Gas



$\sim 5 \times 20 \text{ m}^2$

H. Wang et al, Nat.  
Comm. 6:7459 (2015)

# 3-Step Model

- ① Electron orbit in “continuum”:

$$v_x \sim v_0 \sin(\omega t) \quad a = \begin{cases} 0 & \text{linear polarization} \\ \pm 1 & \text{circular polarization} \end{cases}$$

$$v_y \sim a v_0 \cos(\omega t)$$

For circularly polarized field, the e- never returns in vicinity of the ion



Only **linearly polarized** light is emitted after recombination

- ② Electron orbit is anti-symmetric in the rest frame (“figure-8” electric dipole)

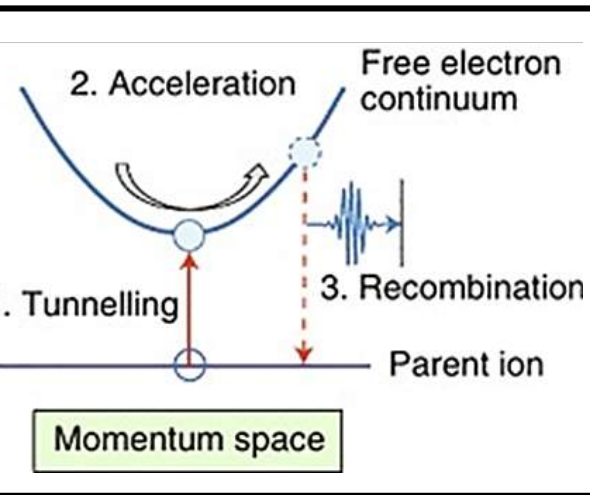
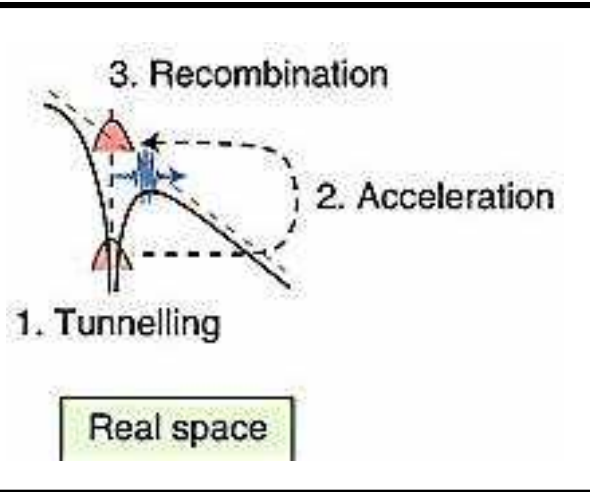


Only **odd harmonics** are allowed

- ③ Electron recombination happens every half-cycle of the laser field



Harmonics are separated by  $2\omega_{\text{laser}}$



L. Ortmann A.S. Landsman, Adv.  
AMO Phys. 70 (2021)



# Photon Energy

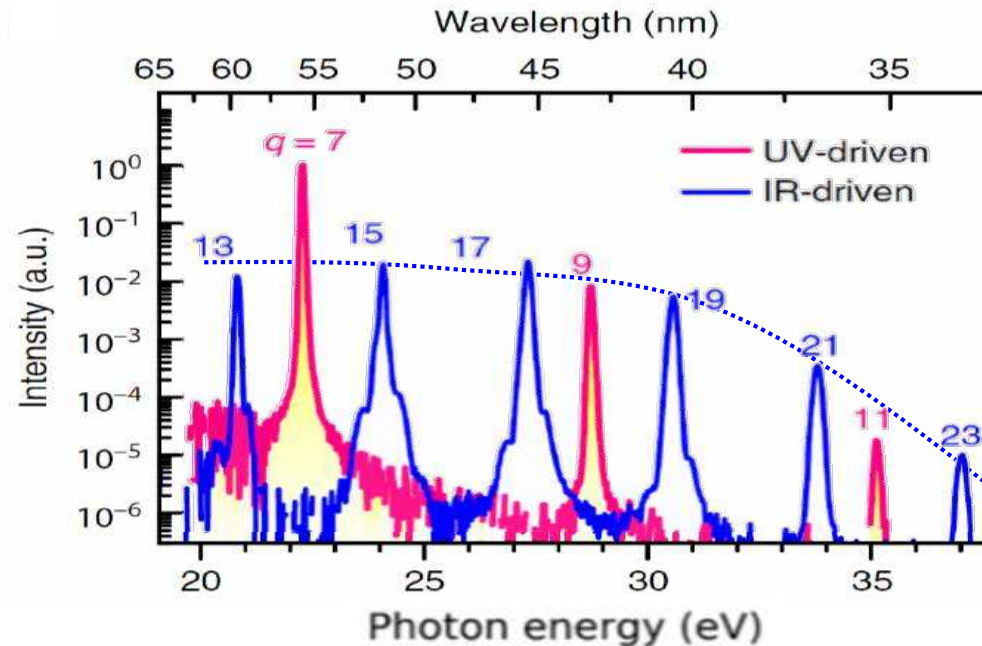
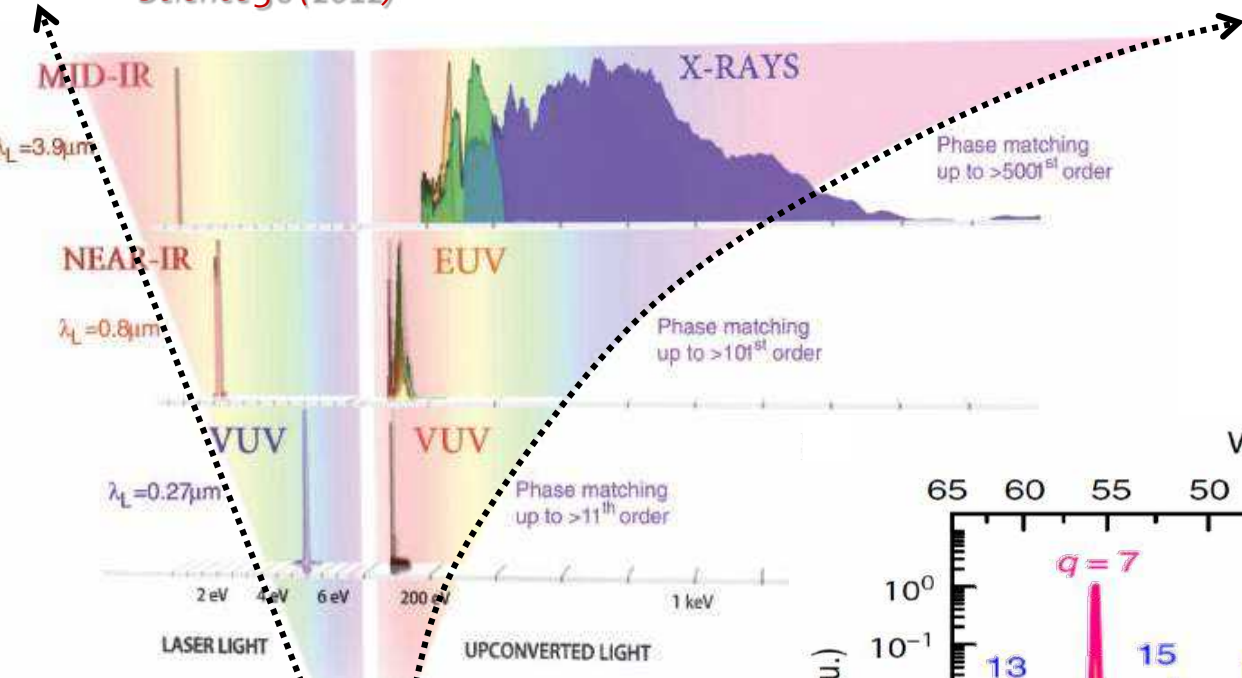
Energy conservation:

$$E_\gamma \leq E_e \text{ at impact + ionization potential}$$

$$E_\gamma \leq 3.17U_p + I_p \text{ cut-off}$$

$$U_p = \frac{(eE_0)^2}{4m_e\omega^2} \text{ ponderomotive potential energy}$$

T. Popminchev et al., Science 36 (2012)



Yb, 1030 nm,  
 ~0.5 MV/cm or 10<sup>13</sup> W/cm<sup>2</sup>  
 I<sub>p</sub> (He) = 25 eV  
 ⇒ E<sub>γ</sub> < 130 eV

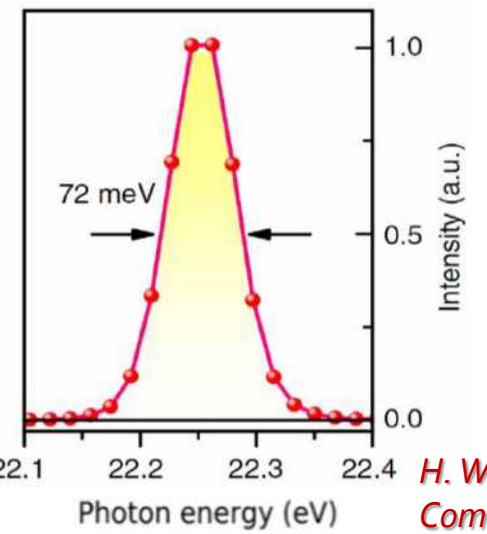
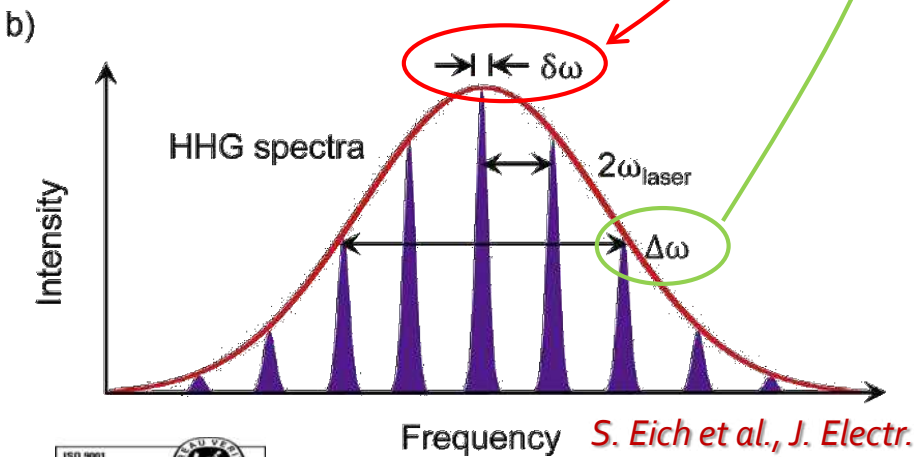
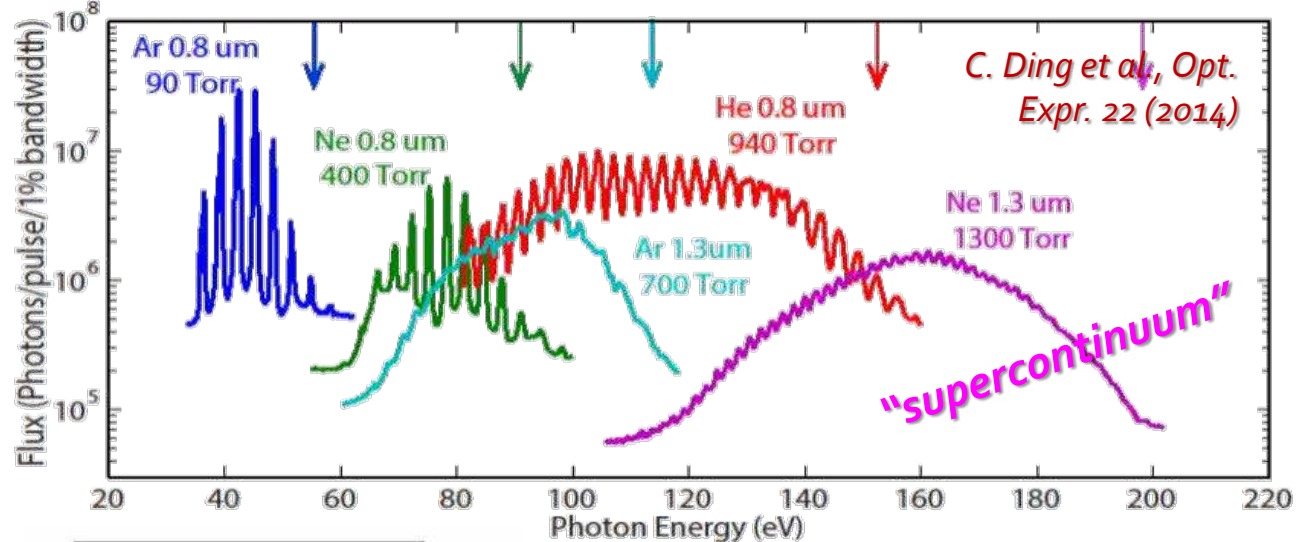
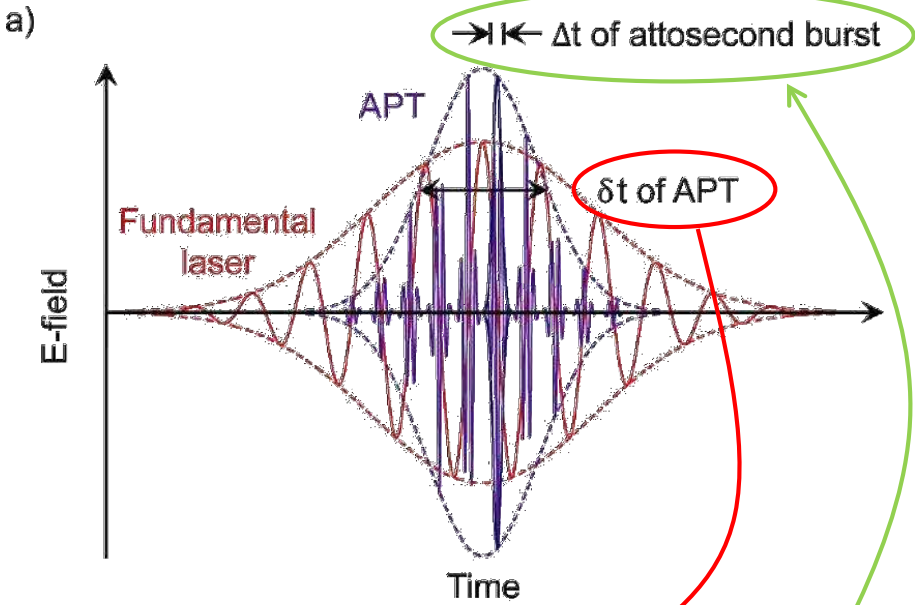
H. Wang et al, Nat. Comm. 6:7459 (2015)





# Spectrum

$$\lambda_L = 0.3 - 1 \mu\text{m} \Rightarrow 2\omega_L \sim 8 - 2 \text{ eV}$$



Ti:Sa, 100 fs  
 $h\nu = 20 \text{ eV}$   
 $\Rightarrow \text{bw} \approx 0.2\%$

*S. Eich et al., J. Electr. Spectr. Rel. Phen. (2014)*

*H. Wang et al., Nat. Comm. 6:7459 (2015)*







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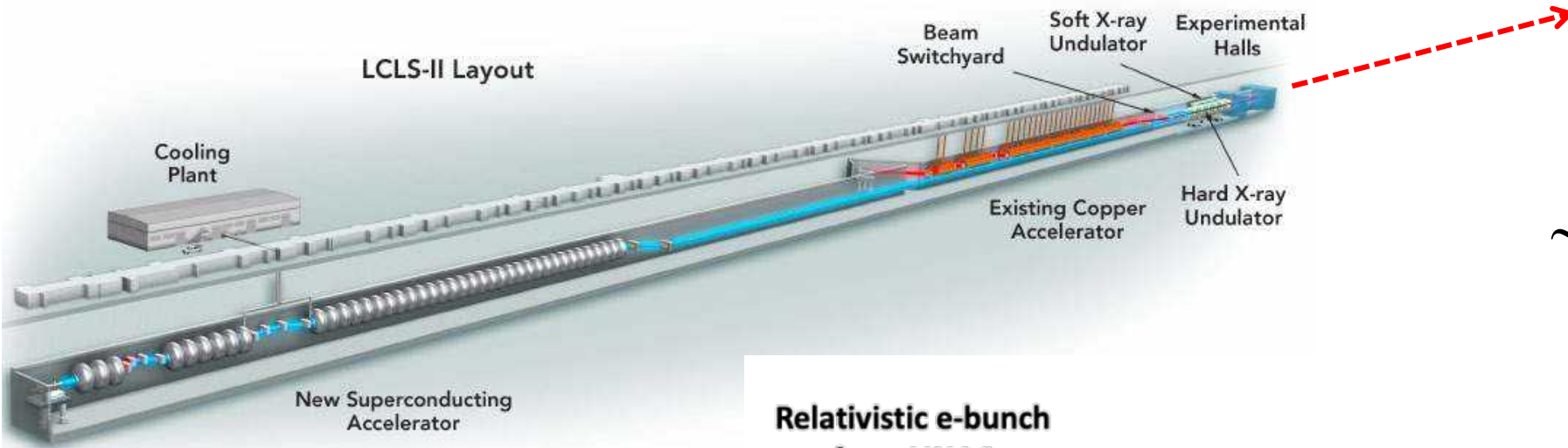
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- Free-Electron Laser (high gain regime)

## ☐ Overview and Perspectives

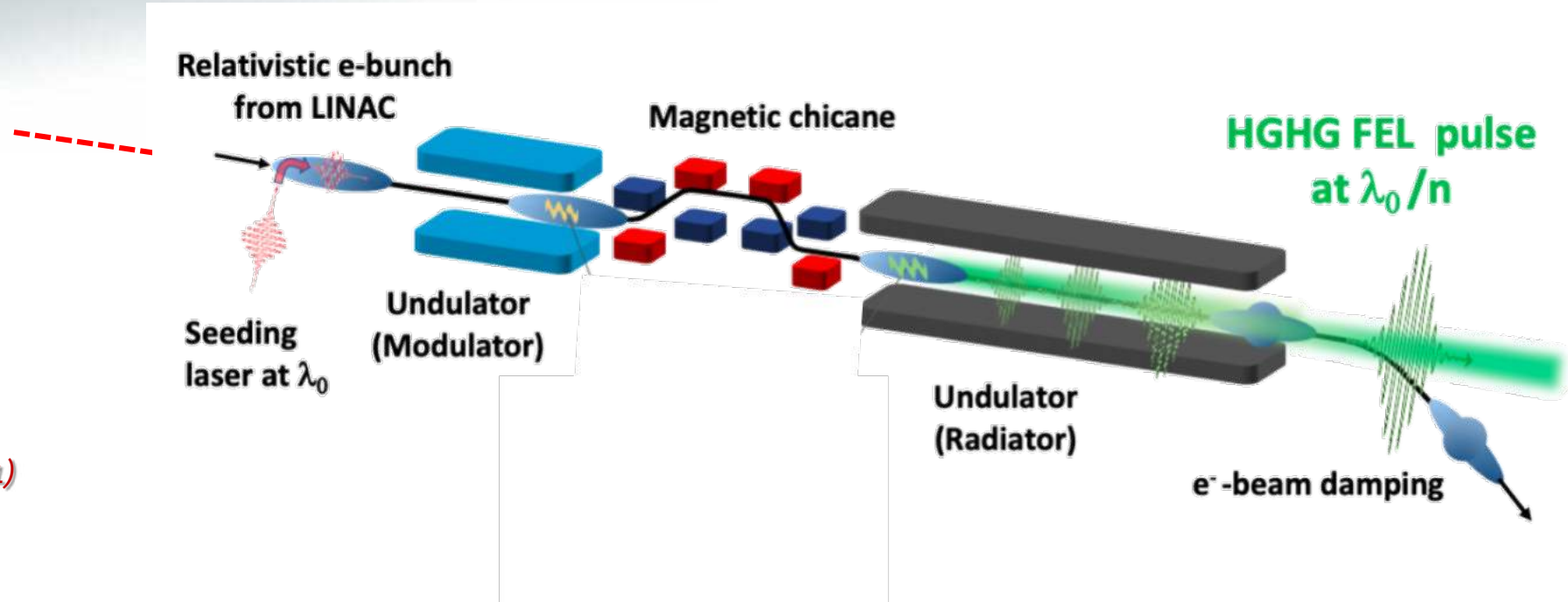


# High Gain FEL

LCLS-II Layout



~ 5 m × 0.1 - 3 km

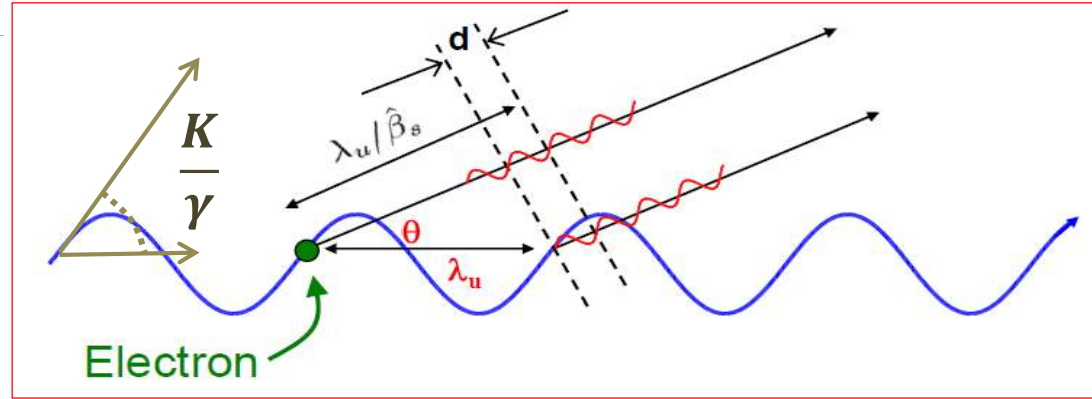
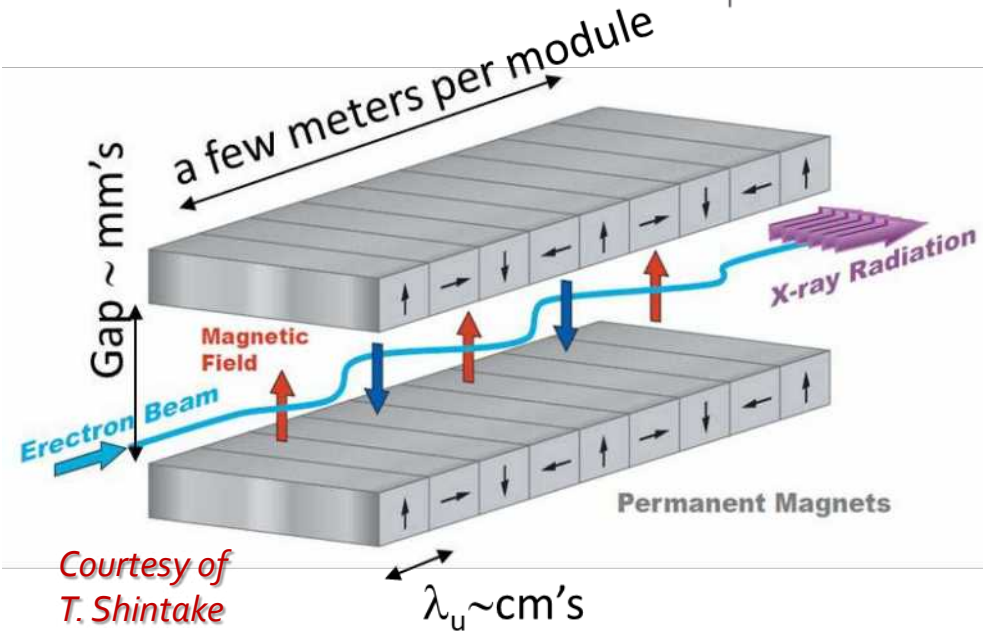


*LCLS-II TDR*  
*F. Benatti et al., Opt. Expr. 29, 24 (2021)*





# Undulator Spontaneous Radiation

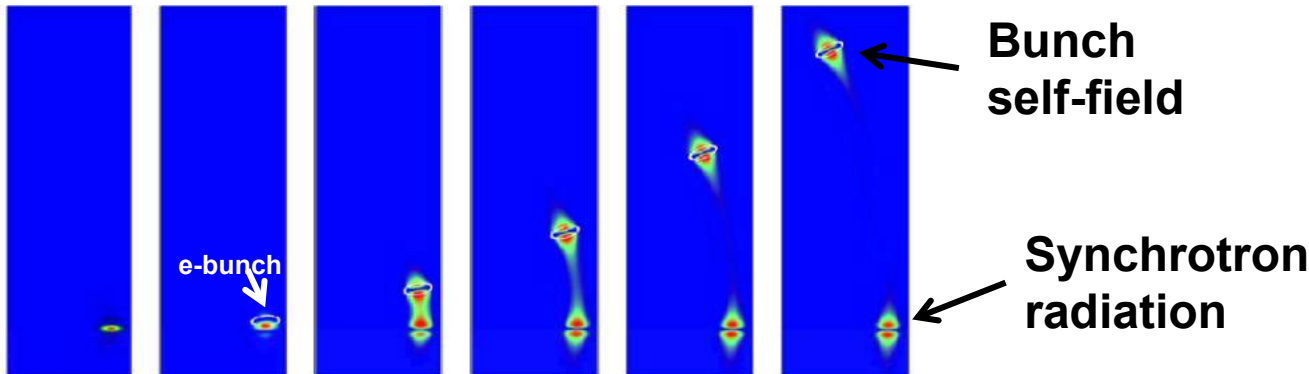


Undulator parameter

$$K = \frac{eB_y \lambda_u}{2\pi m_0 c}$$

Constructive interference:  $\frac{\lambda_u}{v_z} - \frac{\lambda_u \cos \theta}{c} = \frac{\lambda}{c} \Rightarrow \lambda = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$

central wavelength      red-shift off-axis

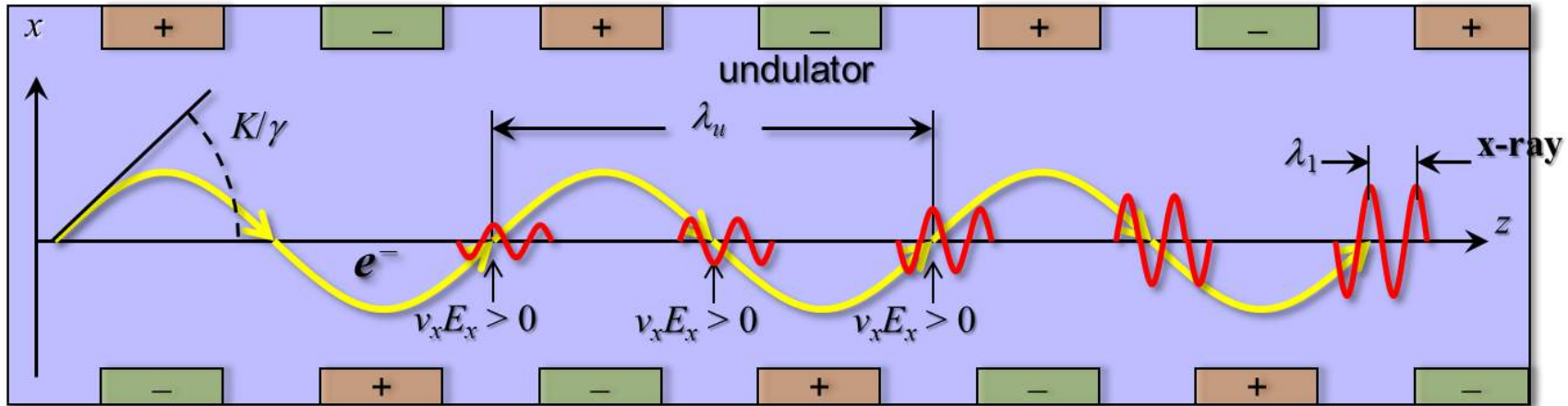


$$P_{SR} \propto \gamma^4 = \frac{E^4}{m_0^4}$$





# Undulator Stimulated Radiation



Energy exchange:

$$P = q\vec{E} \cdot \vec{v}_\perp > 0$$

Ponderomotive phase:

$$\zeta = (k_u + k)z - \omega t$$

Courtesy of  
P. Emma

Synchronization:

$$\frac{d\zeta}{dt} \equiv 0 \Rightarrow v_z = \frac{\omega}{k_u + k} = v_p$$

“Resonance” condition:

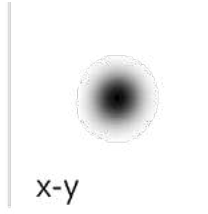
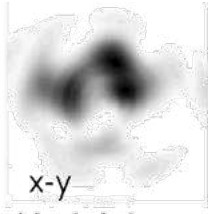
$$\frac{\lambda_u + \lambda}{c} = \frac{\lambda_u}{v_z} \Leftrightarrow \lambda = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right)$$



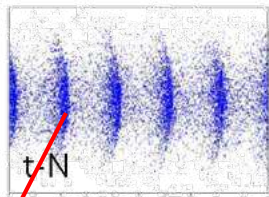
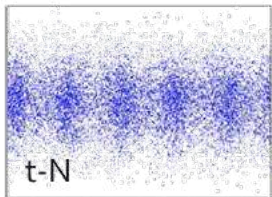
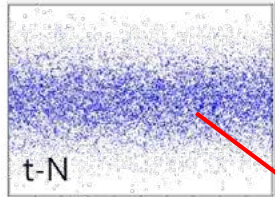
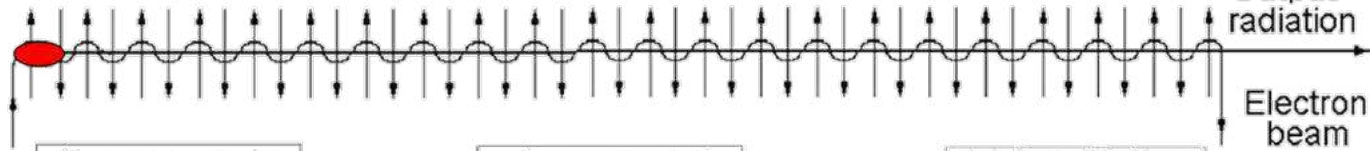


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# High Gain



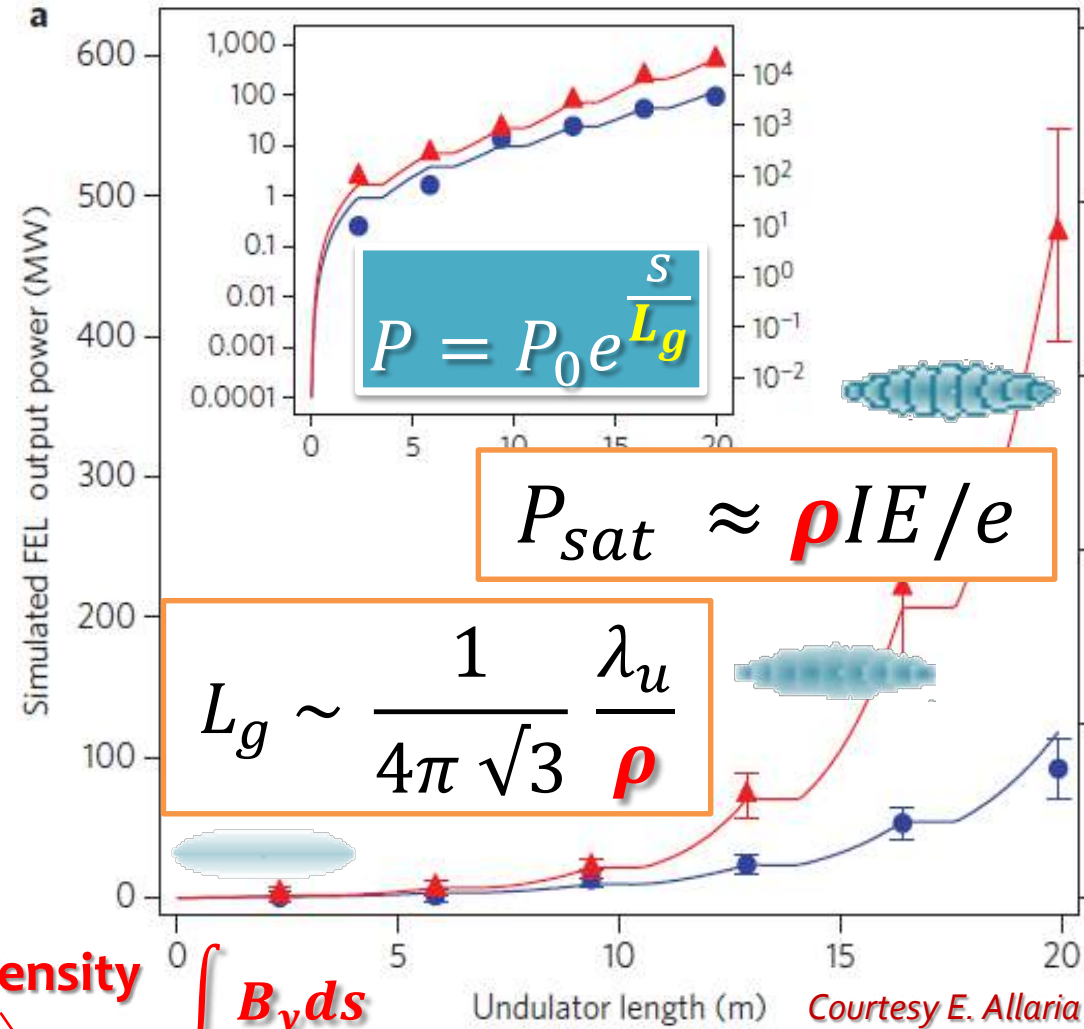
Undulator



t-N

t-N

t-N



3-D charge density  $\int B_y ds$

$$\rho = \frac{1}{\gamma_r} \left[ \frac{\omega_p \lambda_u a_K}{8\pi c} \right]^{\frac{2}{3}}$$

beam energy

Courtesy E. Allaria

Courtesy E. Saldin

$$N_{ph}(\lambda_u) \sim \pi \alpha (N_e + N_e^2)$$

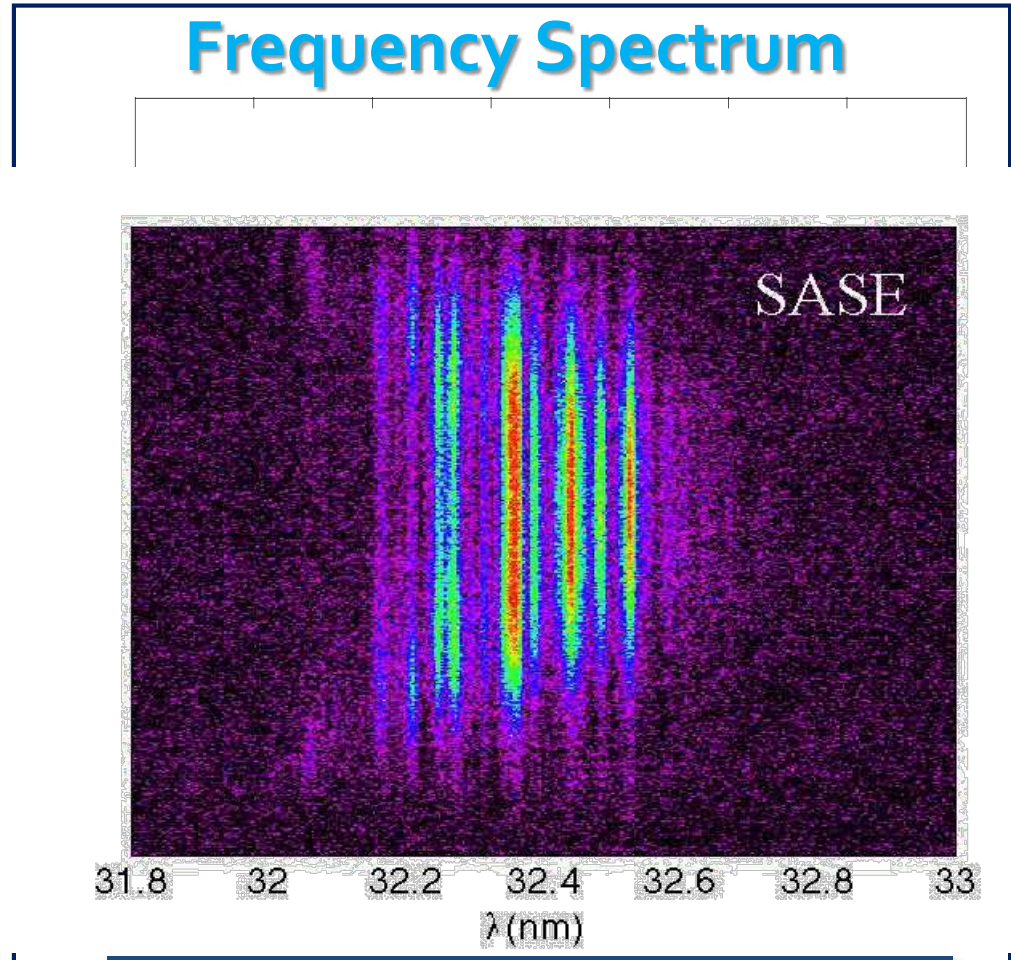
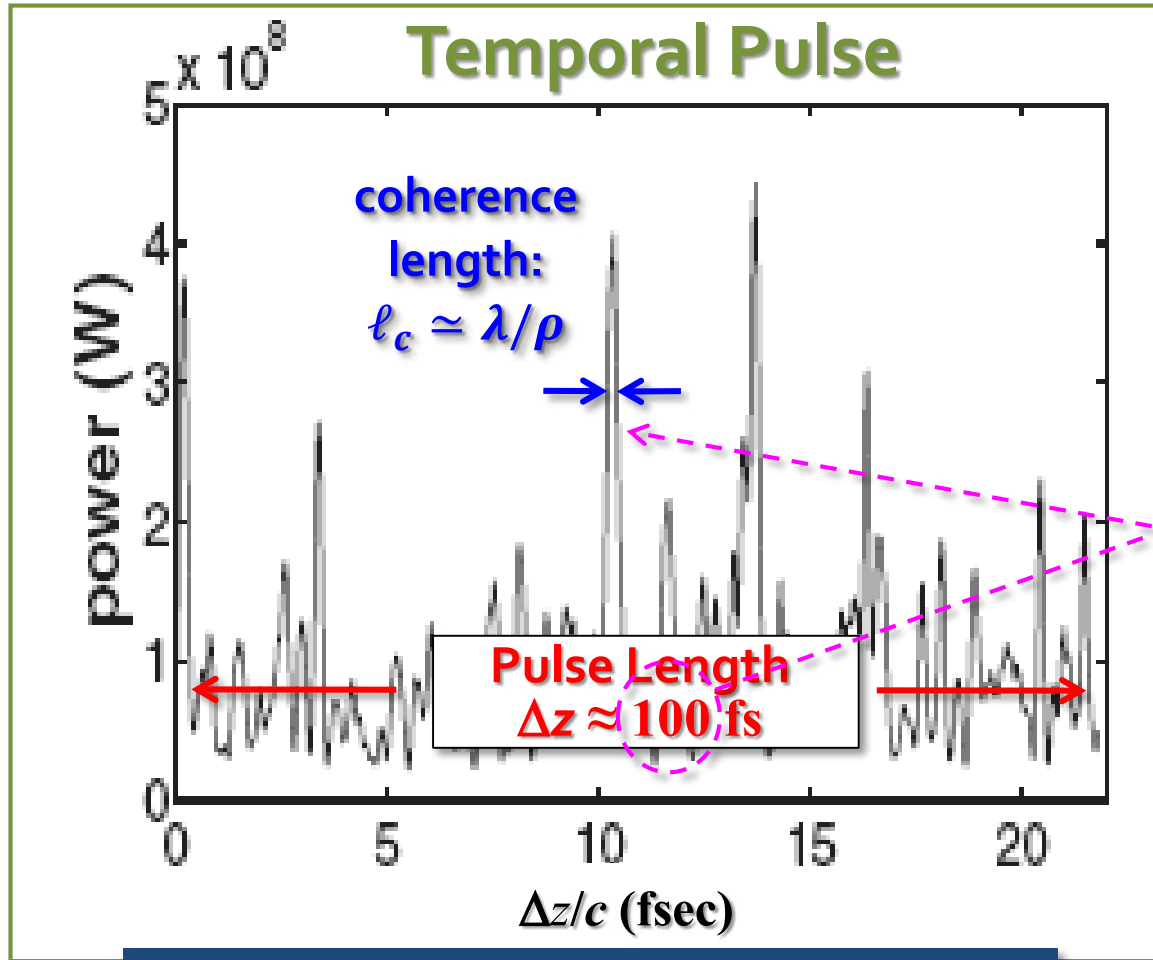




# Self-Amplified Spontaneous Emission

spikes appear in temporal pulse

spikes also in spectrum



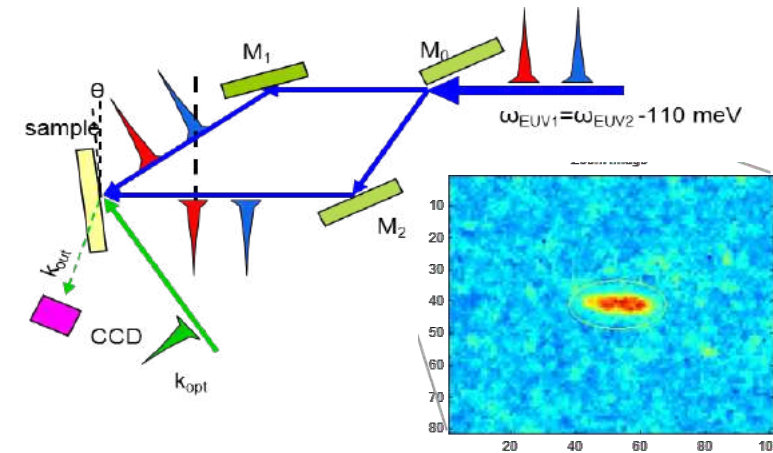
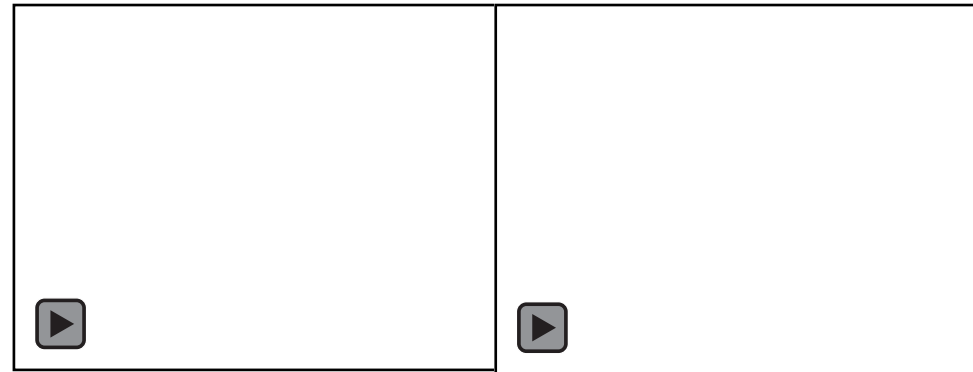
$$\lambda = 1 \text{ \AA}, \rho \approx 1 \times 10^{-3} \rightarrow \ell_c \approx 0.3 \text{ fs}$$

$$\Delta z/c = 100 \text{ fs} \rightarrow \lambda/\Delta z \approx 3 \times 10^{-6}$$



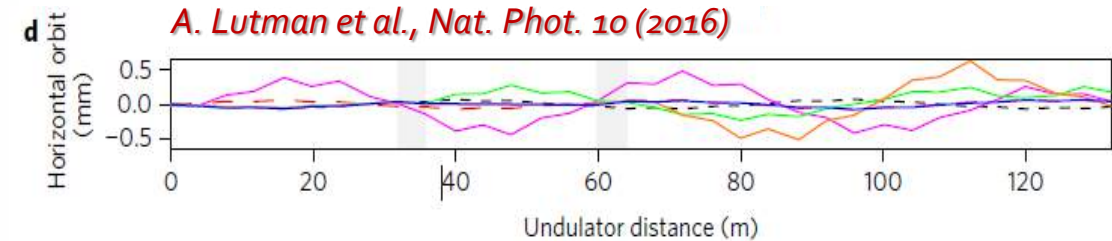
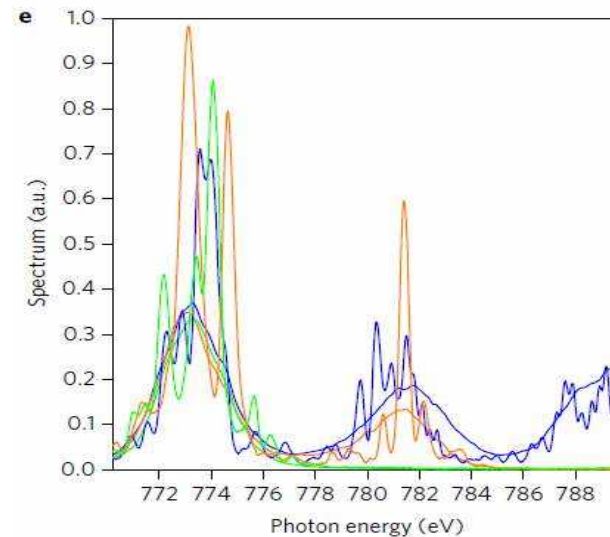
# Operating Modes

2/3-color, pump-probe  
4-wave mixing



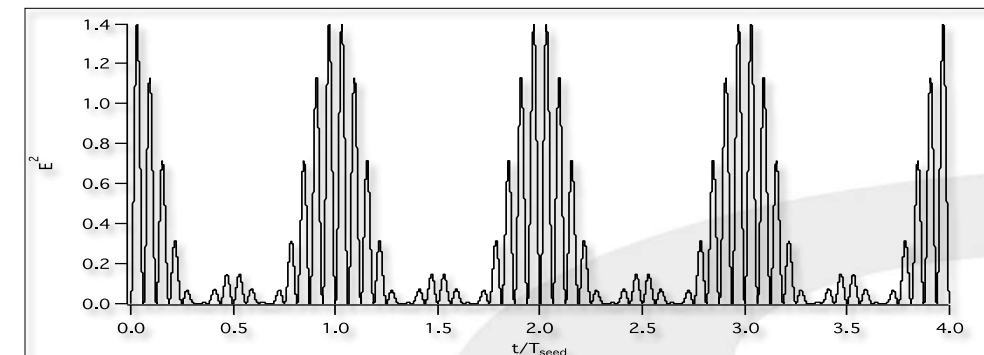
*F. Bencivenga et al., Faraday Discuss. 194 (2016)*

Different colors on  
different orbits



*A. Lutman et al., Nat. Phot. 10 (2016)*

The superposition of >3 phase-locked FEL  
harmonics generates a train of attosecond pulses



*G. Sansone et al., Nature 578, 386 (2020)*



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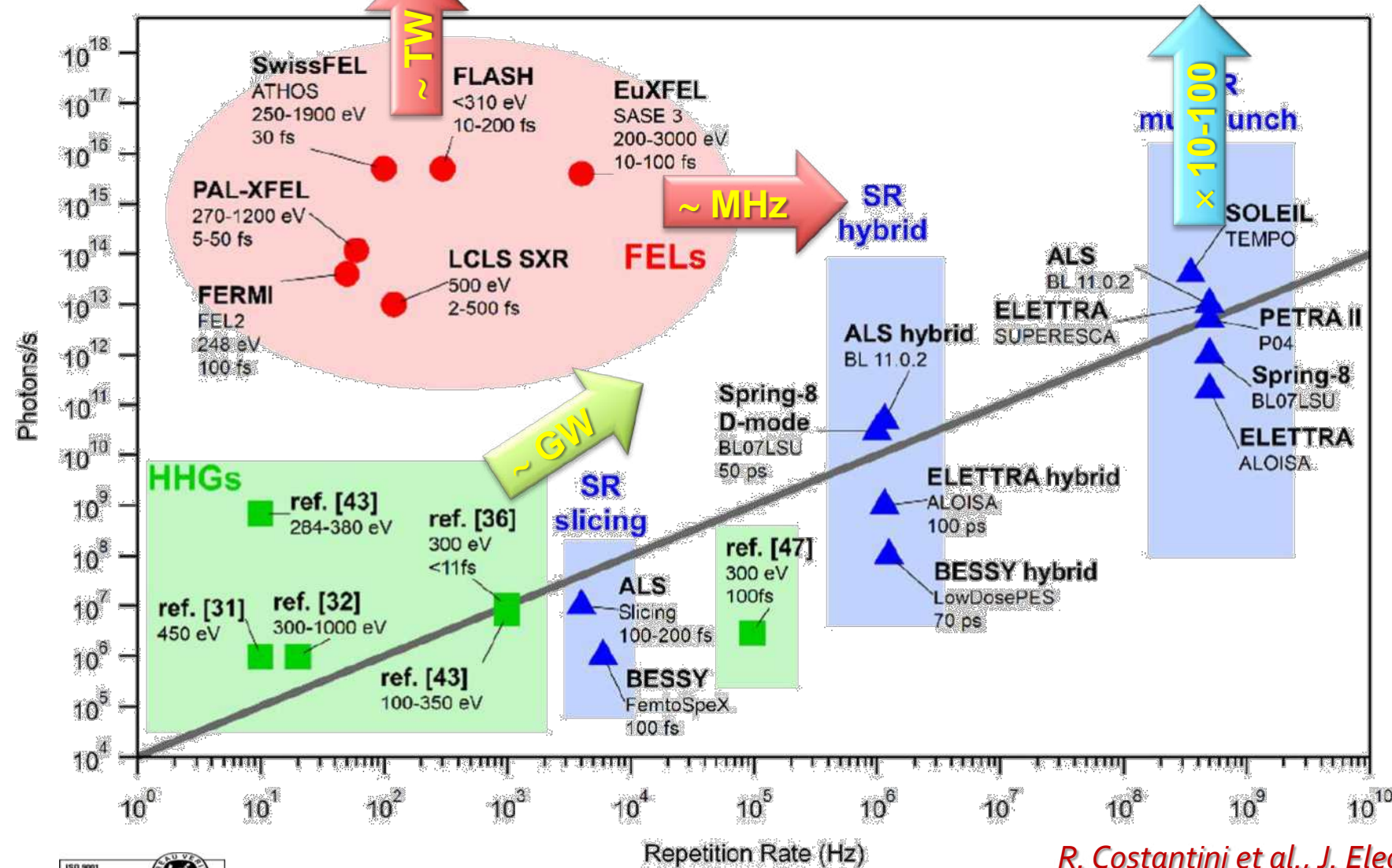
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## ☐ Overview and Perspectives



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# Average Pulse Energy



## SRLS:

- ✓ easyness of access
- ✗ lack (sub-)ps pulses

## HHG:

- ✓ lab-scale size
- ✗ lack pulse energy

## FEL:

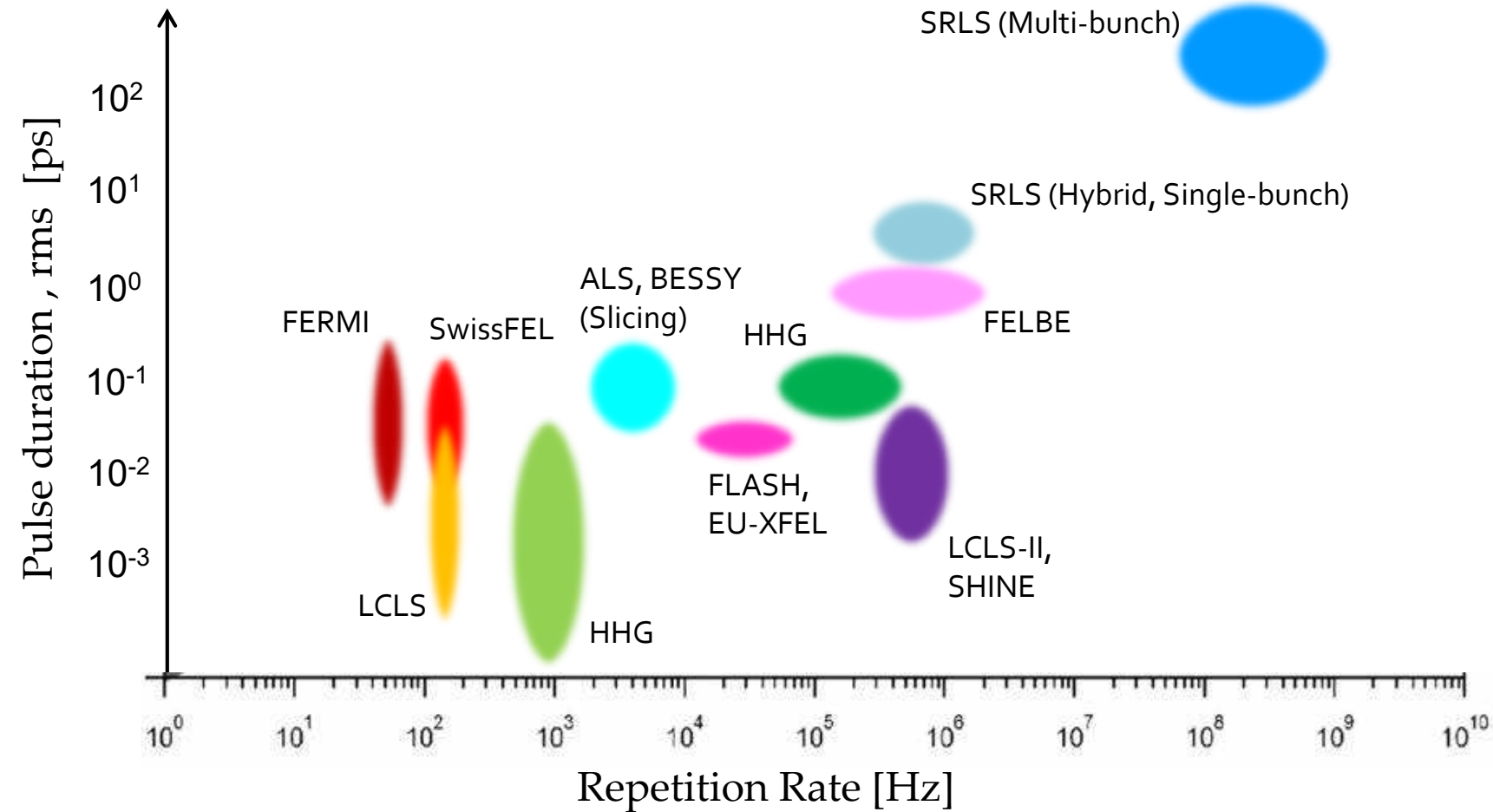
- ✓ large peak power
- ✗ limited access

R. Costantini et al., *J. Electr. Spectr. Rel. Phen.* 254 (2022)





# Pulse Duration



## SRLS:

new generation (multi-bend) tends to produce **longer bunches**

## HHG:

pushing **<fs** pulses towards **MHz** repetition rate

## FEL:

super-conducting linacs target **as** pulses at **MHz** repetition rate

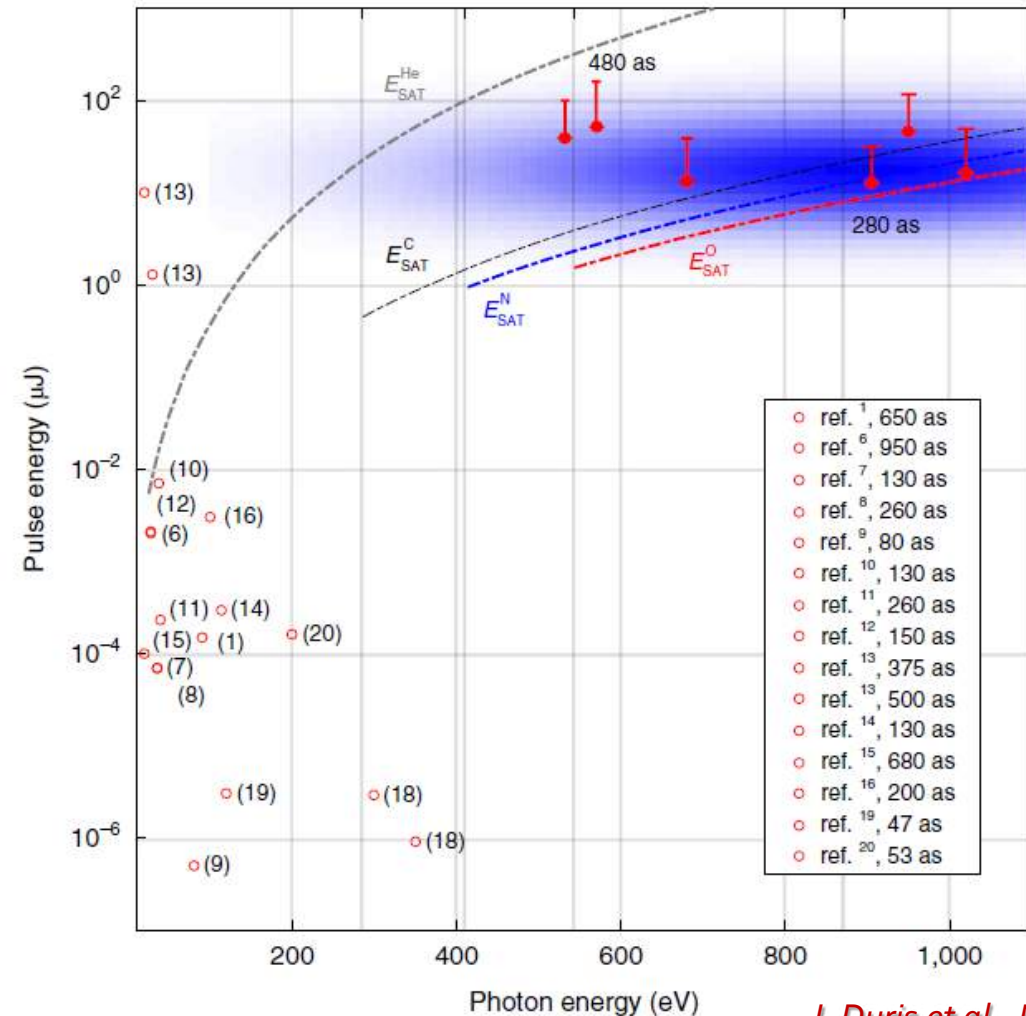




# Peak Pulse Energy

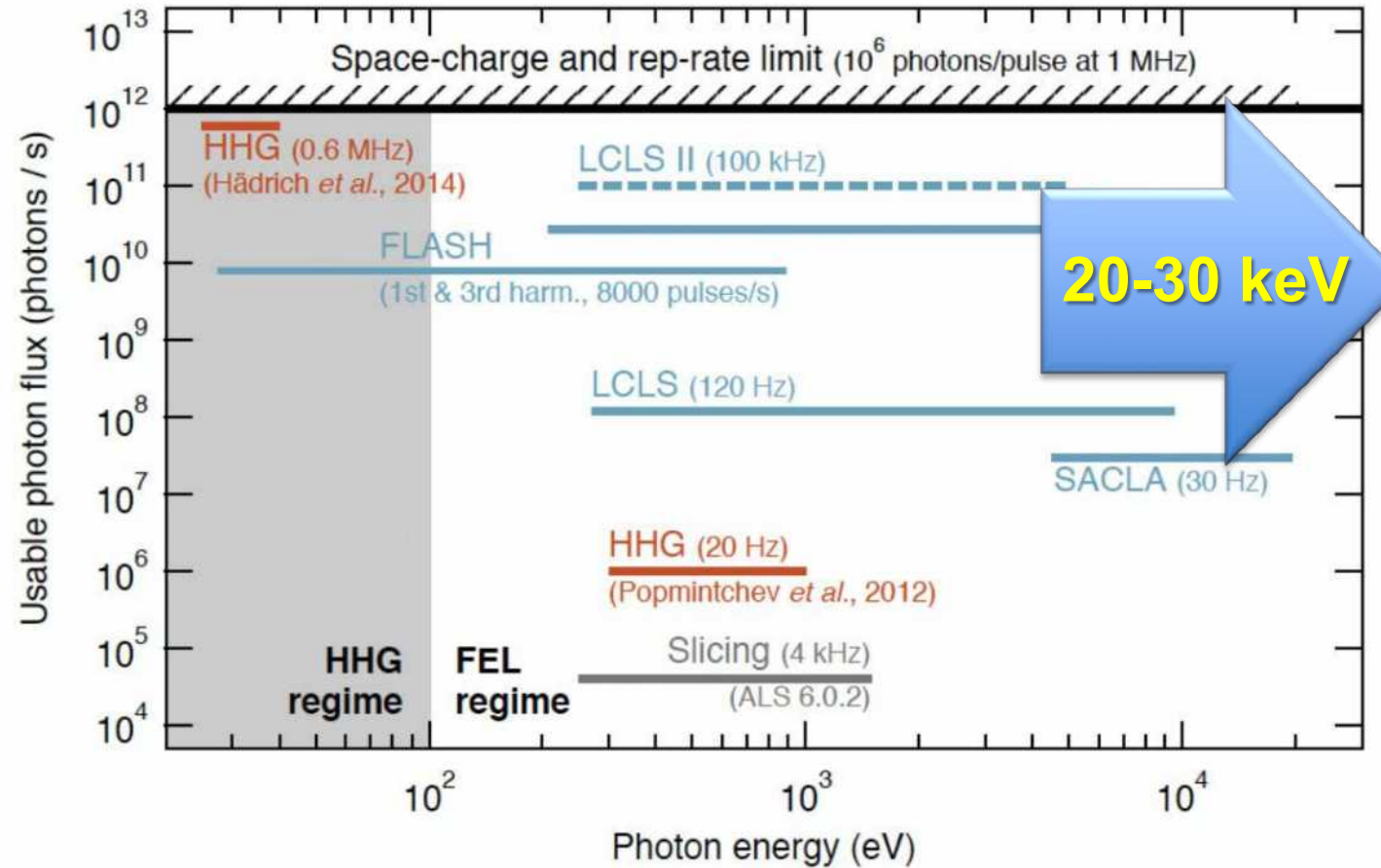
K-shell absorption edge

C N O Ne



*J. Duris et al., Nat. Phot. 14 (2020)*

*Courtesy S.L. Molodtsov, School on SR & FEL Methods (2018)*

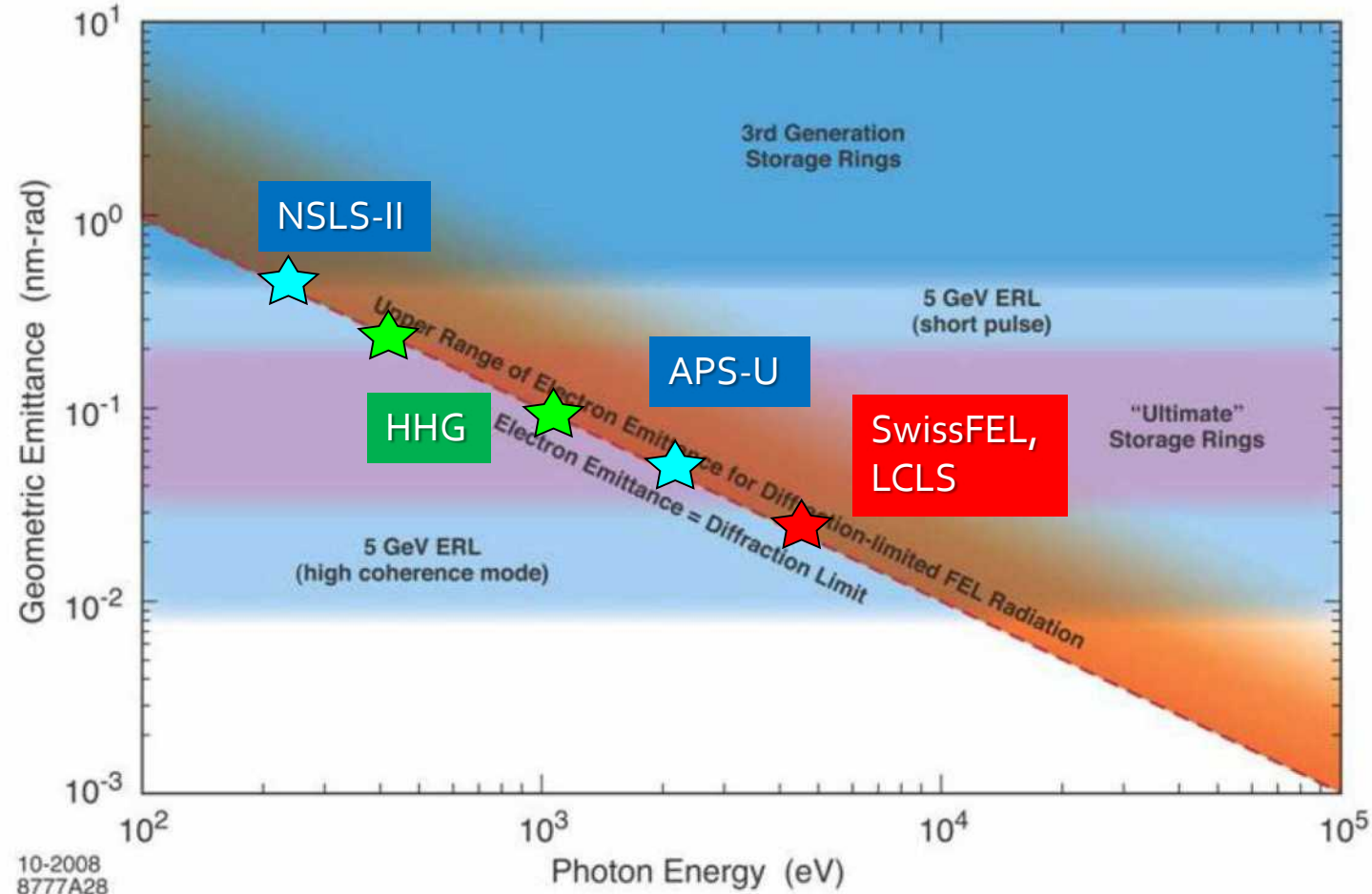
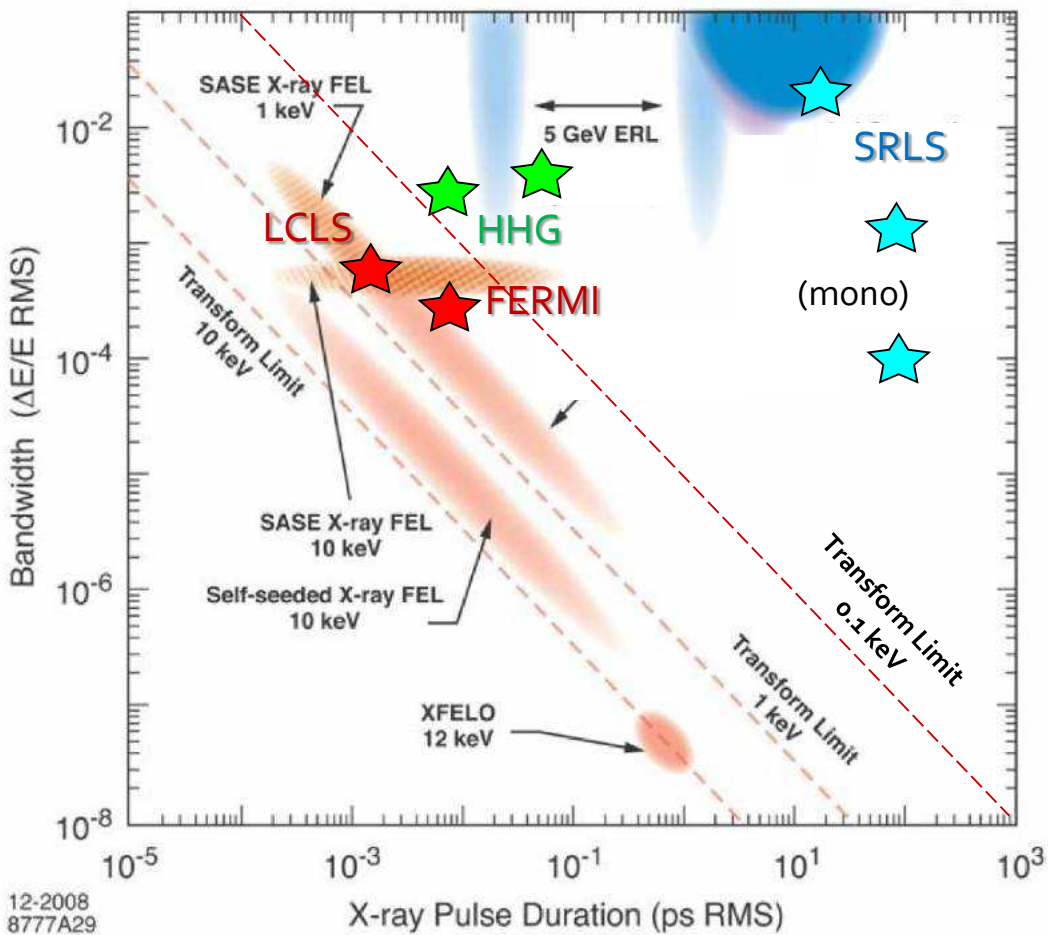




# Coherence

**Fourier Transform Limit,  $\sigma_v \sigma_t = \frac{1}{4\pi}$**

**Diffraction Limit,  $\epsilon_{x,y} = \frac{\lambda}{4\pi}$**



12-2008  
8777A29

10-2008  
8777A28

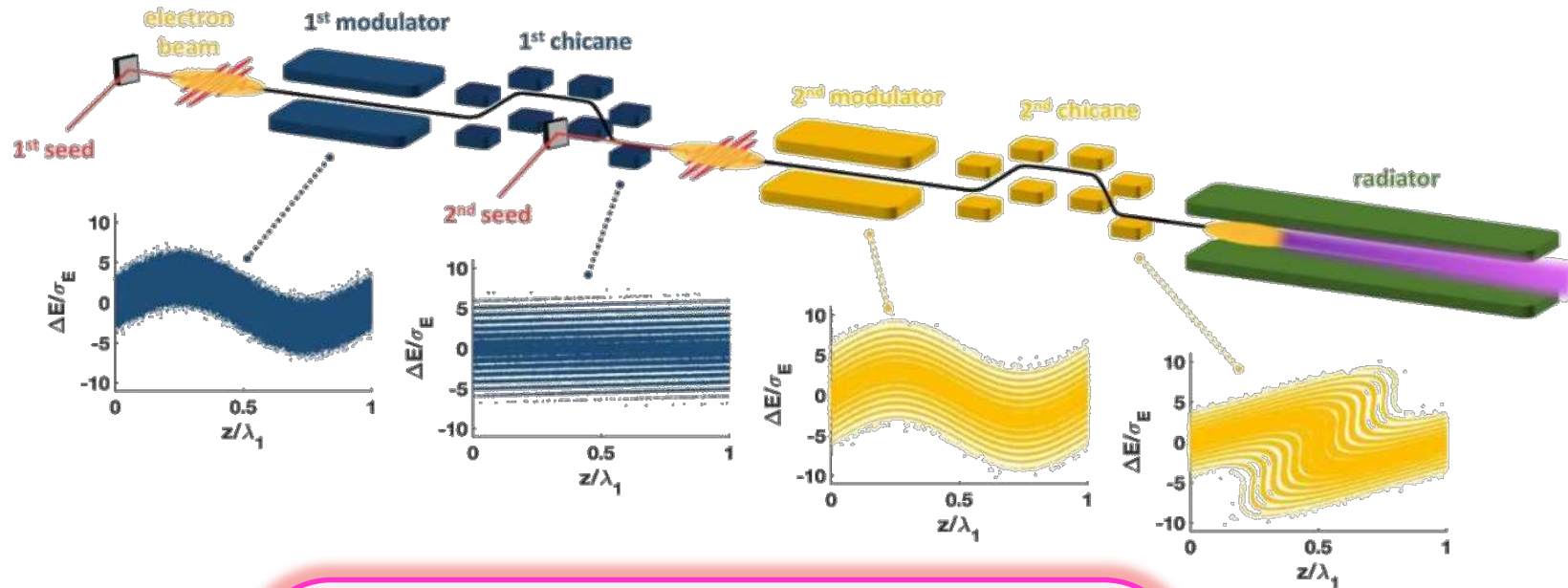






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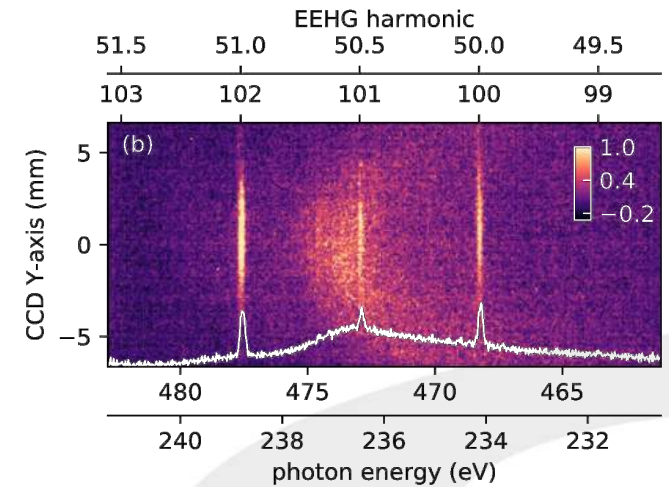
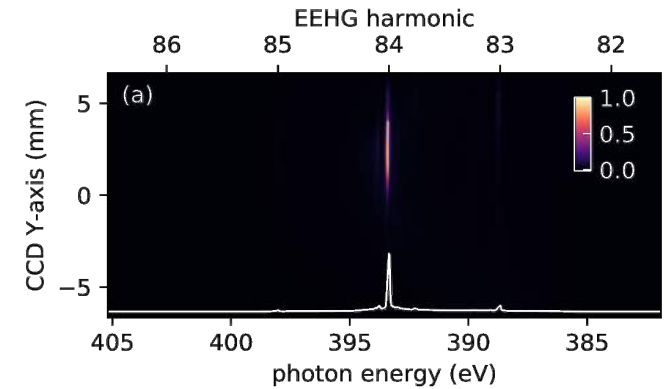
# EEHG FEL



*P. R. Ribic et al.,  
Nat. Phot. 13 (2019)*

## Echo-Enabled Harmonic Generation:

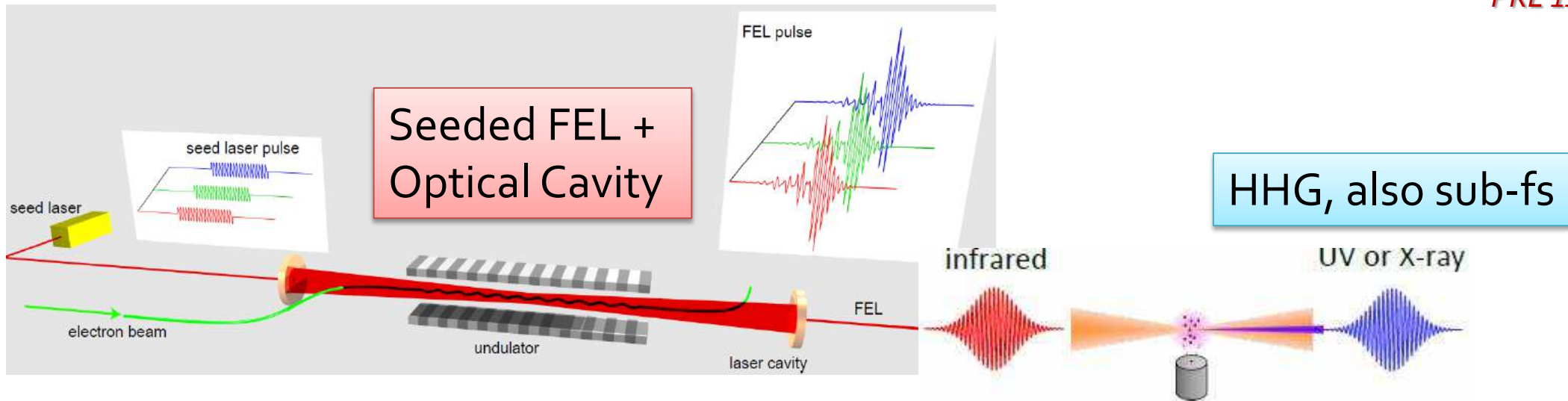
- stable, intense, fully coherent pulses at  $\sim 1$  keV
- *but rep. rate*  $< 1$  kHz





# CEP-FEL HHG

R. Hajima, R. Nagai  
PRL 119 (2017)



## FEL-HHG:

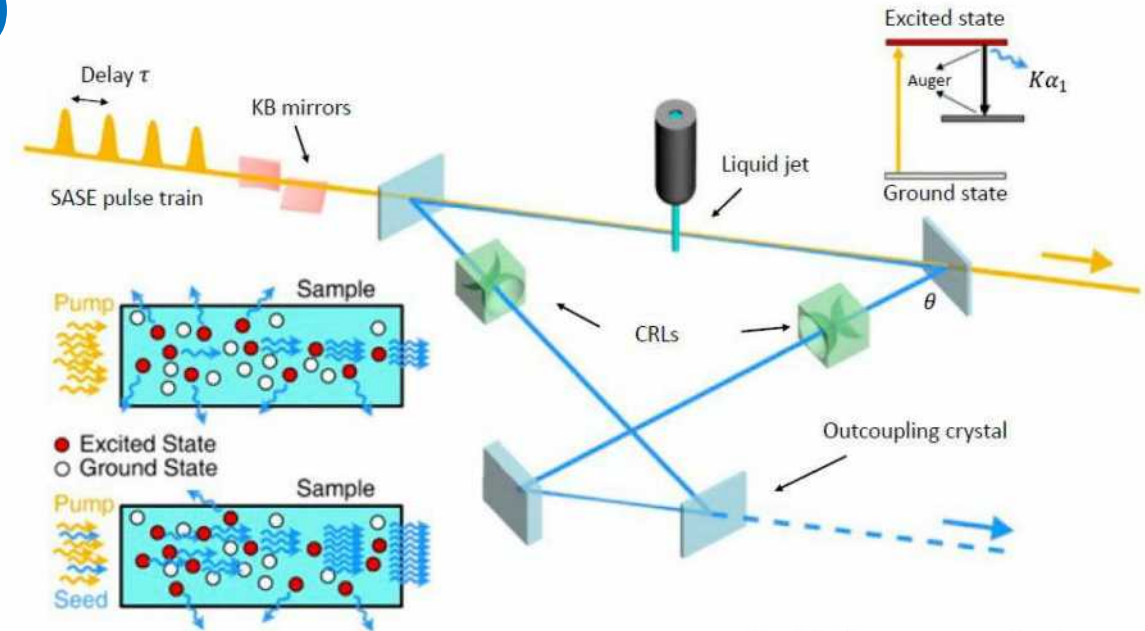
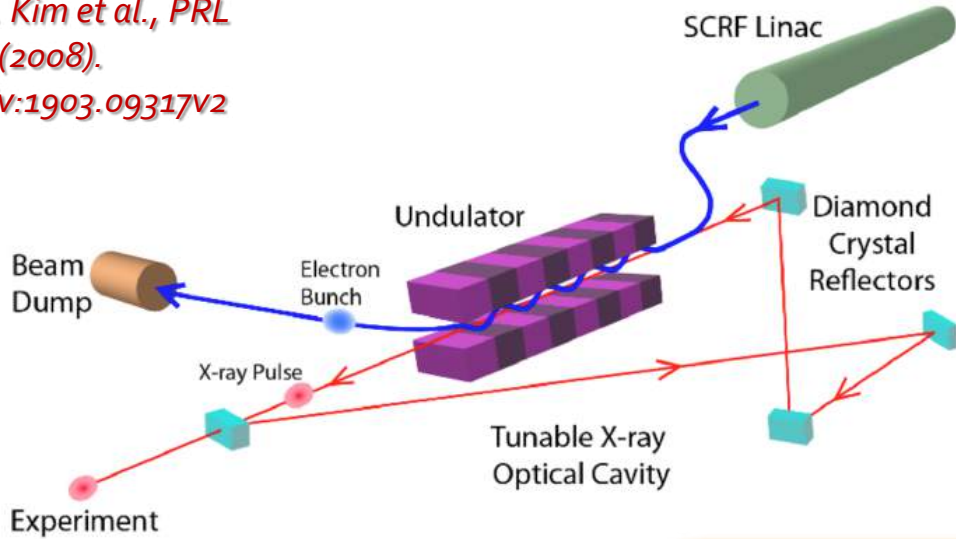
- stable, fully coherent pulses at  $\sim 1\text{--}10$  keV, (sub-)fs,  $< 1$  MHz
- $20 \times 50$  m<sup>2</sup> size
- *but*  $< 10$ 's  $\mu$ J pulse energy



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# XFELO, XLO

*K.-J. Kim et al., PRL  
100 (2008).  
arXiv:1903.09317v2*



*A. Halavanau et al.,  
PNAS 117, 27 (2020)*

**Table 2. Comparison of some XLO and XFELO parameters at LCLS-II**

| Parameter                             | XLO                | XFELO       |
|---------------------------------------|--------------------|-------------|
| Gain per pass                         | Up to $10^6$       | 1.2 to 1.5  |
| Cavity length, m                      | $\sim 10$          | $> 260$     |
| Lasing medium size, m                 | $3 \times 10^{-4}$ | $\sim 100$  |
| Angular tolerance, $\mu\text{rad}$    | 1                  | $\sim 0.01$ |
| Number of photons (max)               | $5 \times 10^{10}$ | $10^{10}$   |
| Peak power, MW                        | $\sim 270$         | $\sim 4.7$  |
| Pulse length, fs                      | 37.4               | 530         |
| FWHM $\Delta t \Delta \omega$ , fs-eV | 1.8                | 4.4         |

FWHM, full-width at half-maximum.

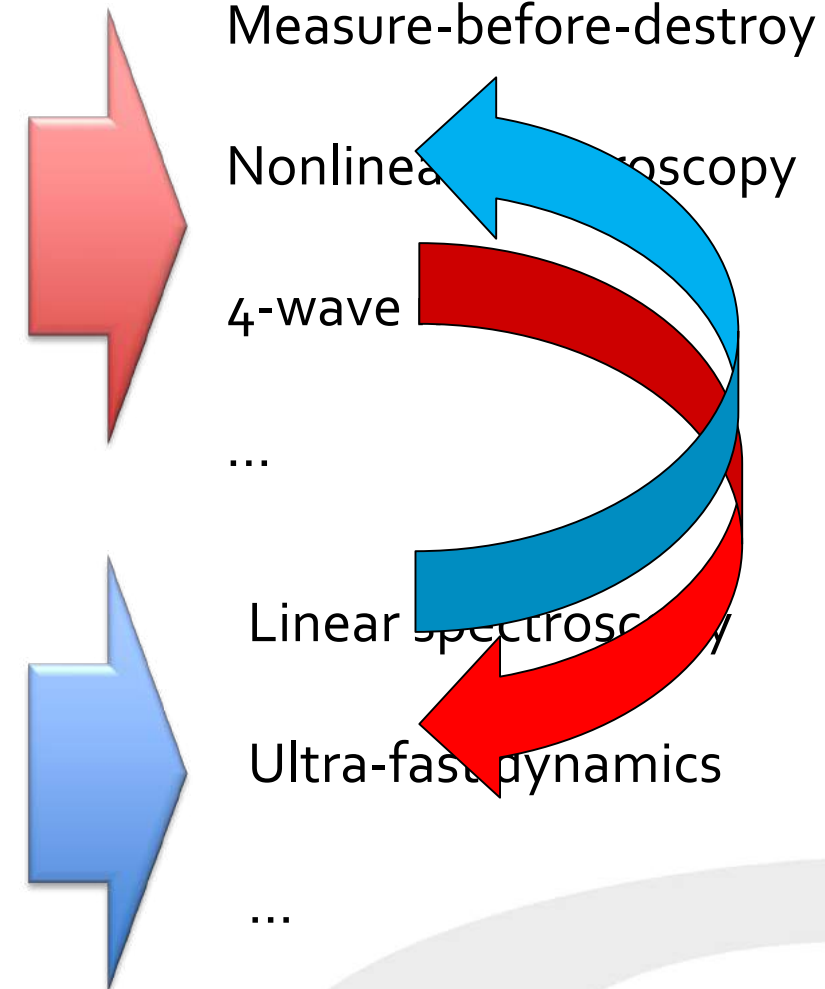


## Present XUV FELs:

- Large size, > 100 – 1000 M€ -scale, limited access
- Low/burst repetition rate
- ~100% transversally coherent, high power density light pulses
- ~100% longitudinally coherent pulses in UV or hard x-ray
- Multi-color, 2-pulse, continuous  $\lambda$  & polarization control

## Present HHGs:

- Small size, 1 – 10 M€ -scale, easy access
- High, tuneable repetition rate
- Fully coherent light pulses at moderate power density over entire XUV range
- Limited  $\lambda$  & polarization control





Elettra  
Sincrotrone  
Trieste

Thank you for your kind attention

*Questions and comments are welcome!*

